# Real numbers as CF's

**EG** rational numbers

$$\frac{19}{5} = [3; 1, 4]$$

$$19 = 3 \cdot 5 + 4$$
$$5 = 1 \cdot 4 + 1$$
$$4 = 4 \cdot 1 + 0$$

$$= [3; 1, 3, 1]$$

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cdots}}$$

$\frac{5}{4}$

**EG** irrational numbers

$$e = [2; 1, 2, 1, \ldots]$$

$e = 2.7182\ldots$

$$1/0.7182\ldots = 1.3922\ldots$$
$$1/0.3922\ldots = 2.5496\ldots$$
$$1/0.5496\ldots = 1.8194\ldots$$

$$2 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cdots}}$$

$\frac{1}{0.7182\ldots}$

would terminate if: $1/0.\ldots$ = integer

**THM** Every real number $x$ can be represented as a simple CF.

- if $x$ is irrational: the CF for $x$ is unique and infinite
- if $x$ is rational: there are exactly 2 CFs for $x$; both are finite (one ends in 1)