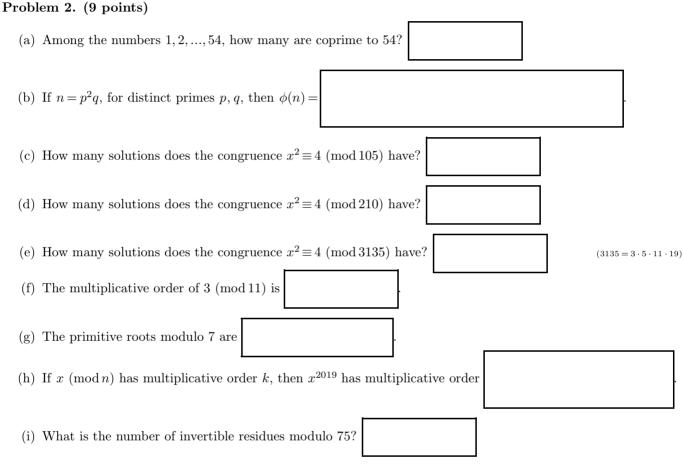
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Problem 1. (3 points) What is the last (decimal) digit of 7¹²³⁴⁵⁶?

Solution. We need to determine $7^{123456} \pmod{10}$. Since $\gcd(7, 10) = 1$ and $\phi(10) = \phi(2)\phi(5) = 4$ and $123456 \equiv 56 \equiv 0 \pmod{4}$, we have $7^{123456} \equiv 7^0 \equiv 1 \pmod{10}$. This means that the last (decimal) digit of 7^{123456} is 1.



Solution.

- (a) $\phi(54) = \phi(2)\phi(27) = 27 9 = 18$
- (b) If $n = p^2 q$, for distinct primes p, q, then $\phi(n) = \phi(p^2)\phi(q) = (p^2 p)(q 1)$.
- (c) By the CRT, since $105 = 3 \cdot 5 \cdot 7$, the congruence has $2 \cdot 2 \cdot 2 = 8$ solutions.
- (d) By the CRT, since $210 = 2 \cdot 3 \cdot 5 \cdot 7$, the congruence has $1 \cdot 2 \cdot 2 \cdot 2 = 8$ solutions. (Note that $x^2 \equiv 4 \pmod{2}$ only has one solution; namely, $x \equiv 0$.)
- (e) By the CRT, since $3135 = 3 \cdot 5 \cdot 11 \cdot 19$, the congruence has $2 \cdot 2 \cdot 2 \cdot 2 = 16$ solutions.
- (f) The multiplicative order of 3 (mod 11) is 5.
- (g) The primitive roots modulo 7 are 3, 5.
- (h) If x (mod n) has multiplicative order k, then x^{2019} has multiplicative order $\frac{k}{\gcd(k, 2019)}$.
- (i) $\phi(75) = \phi(3)\phi(25) = 2 \cdot 20 = 40$