Midterm #1: practice

MATH 311 — Intro to Number Theory
midterm: Tuesday, Sep 20

Please print your name:

Problem 1. Using induction, prove that $1^2+3^2+5^2+\ldots+(2n+1)^2=\frac{(n+1)(2n+1)(2n+3)}{3}$ for all integers $n\geqslant 0$.

Problem 2. Which are the possible remainders that the square of an integer leaves upon division by 6?

Problem 3. Find $d=\gcd(119,272)$. Using the Euclidean algorithm, find integers x,y such that $119x+272y=d$.

Problem 4.

(a) For which values of k has the diophantine equation $123x+360y=k$ at least one integer solution?

(b) Determine all solutions of $123x+360y=99$ with x and y positive integers.

Problem 5. We call (a,b,c) a prime triple if a,b,c are all primes.

(a) List the first few prime triples of the form $(p,p+2,p+6)$.
(It is believed, but nobody can show, that there are infinitely many such triples.)

(b) Show that there is only a single prime triple of the form $(p,p+2,p+4)$.

(c) Show that there are no prime triples of the form $(p,p+2,p+5)$.

Problem 6. Using induction, prove that $5|(3^{3n+1}+2^{n+1})$ for all integers $n\geqslant 0$.

Problem 7. Determine, and prove, a formula for the sum $\sum_{k=0}^{n}(4k+1)$.

Problem 8. Using induction, show that any two consecutive Fibonacci numbers are relatively prime. In other words, show that $\gcd(F_n,F_{n+1})=1$ for all integers $n\geqslant 1$.

Armin Straub
straub@southalabama.edu