Homework #3

MATH 311 — Intro to Number Theory
due in class on Thursday, Sep 15

Please print your name:

These problems are not suited to be done last minute!
Also, if you start early, you can consult with me if you should get stuck.

Problem 1.

(a) Find $d = \gcd(100, 2016)$. Using the Euclidean algorithm, find integers x, y such that $100x + 2016y = d$.

(b) Find $d = \gcd(100, 2017)$. Using the Euclidean algorithm, find integers x, y such that $100x + 2017y = d$.

Problem 2.

(a) For which values of k has the diophantine equation $24x + 138y = k$ at least one integer solution?

(b) Determine all integer solutions of $24x + 138y = 18$.

Problem 3. The neighborhood theater charges \$1.80 for adult admissions and \$.75 for children. On a particular evening the total receipts were \$90. Assuming that more adults than children were present, how many people attended?

Problem 4.

(a) Show that $(2, 3)$ is the only pair (p_1, p_2) of primes such that $p_2 = p_1 + 1$.

(b) A pair of primes (p_1, p_2) is a twin prime pair if $p_2 = p_1 + 2$. Show that every twin prime pair except $(3, 5)$ is of the form $(6n - 1, 6n + 1)$.

[*Hint:* Write the pair as $(N - 1, N + 1)$ and think about the possible remainders of N upon division by 6.]

(c) Show that $(2, 5)$ is the only pair (p_1, p_2) of primes such that $p_2 = p_1 + 3$.

(d) Write down a few pairs (p_1, p_2) of primes such that $p_2 = p_1 + 4$.

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