Preparing for Midterm #2

MATH 316 — Linear Algebra II
Wednesday, Mar 27

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 4 & -8 \\ -1 & 6 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

Problem 2.

(a) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

(b) Solve the original differential equation by solving the system.

Problem 3.

(a) What are the possible Jordan normal forms of a 6×6 matrix with eigenvalues $7, 7, 3, 3, 3, 3$?

(b) How many different Jordan normal forms are there for a 10×10 matrix with eigenvalues $8, 6, 6, 2, 2, 2, 1, 1, 1, 1$?

Problem 4. Consider the sequence a_n defined by $a_{n+2} = 4a_{n+1} - a_n$ and $a_0 = 1$, $a_1 = 0$.

(a) Determine the next three terms.

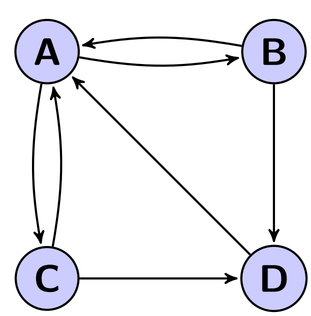
(b) A Binet-like formula for a_n is $a_n = \boxed{}$.

(c) $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{}$.

Again, work on the extra sheet and be sure to check your answer to (b) by comparing with the values in (a).

Problem 5. Suppose the internet consists of only the four webpages A, B, C, D which link to each other as indicated in the diagram.

Rank these webpages by computing their PageRank vector.



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Problem 6. Determine an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

(a) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2\\1\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix}$.

(b) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\1\\1 \end{bmatrix}$.

Problem 7.

(a) The eigenvalues of a 5×5 matrix for orthogonally projecting onto a 3-dimensional subspace are $\boxed{}$.

What are the eigenspaces of that matrix?

(b) Suppose A is the 3×3 matrix of a reflection through a plane (containing the origin).

Then $\det(A) = \boxed{}$, and the eigenvalues of A are $\boxed{}$. What are the eigenspaces of A?

(c) Precisely state the spectral theorem.

(d) If A is a reflection matrix, then $A^{-1} = \boxed{}$.

(e) If $A = \begin{bmatrix} 2 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 4 \end{bmatrix}$, then $A^n = \boxed{}$ and $e^{At} = \boxed{}$.

(f) If $N^4 = \mathbf{0}$, then $e^{Nt} = \boxed{}$.

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