Quiz #1

MATH 237 — Linear Algebra I
Thursday, Aug 25

Please print your name:

Problem 1. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 5 \\ x_1 + 3x_2 + 4x_3 &= 1 \\ 3x_1 - 2x_2 + x_3 &= 3 \end{aligned}$$

(a) Write down the augmented matrix for the system.

(b) Using Gaussian elimination, determine the RREF. (Make sure to record all your row operations!)

(c) From the RREF, read off the solution to the linear system.

Solution.

(a) The augmented matrix is $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 1 & 3 & 4 & 1 \\ 3 & -2 & 1 & 3 \end{array}\right]$.

(b) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 1 & 3 & 4 & 1 \\ 3 & -2 & 1 & 3 \end{array}\right] \overset{\substack{R_2 - R_1 \Rightarrow R_2 \\ R_3 - 3R_1 \Rightarrow R_3}}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & 5 & -4 \\ 0 & -8 & 4 & -12 \end{array}\right] \overset{R_3 + 8R_2 \Rightarrow R_3}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 44 & -44 \end{array}\right]$

$\overset{\frac{1}{44}R_3 \Rightarrow R_3}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 1 & -1 \end{array}\right] \overset{\substack{R_1 + R_3 \Rightarrow R_1 \\ R_2 - 5R_3 \Rightarrow R_2}}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] \overset{R_1 - 2R_2 \Rightarrow R_1}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$

The final matrix is the row-reduced echelon form.

(c) The system has the unique solution $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$.

□

Armin Straub
straub@southalabama.edu