Quiz #6

MATH 237 — Linear Algebra I
Thursday, Oct 15

Please print your name:

Problem 1. Decide whether the following sets of vectors are a basis of $\mathbb{R}^3$. No computations necessary!

(a) $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\4\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}$ ☐ basis of $\mathbb{R}^3$ ☐ not a basis of $\mathbb{R}^3$

(b) $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\4\end{bmatrix}$ ☐ basis of $\mathbb{R}^3$ ☐ not a basis of $\mathbb{R}^3$

(c) $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\4\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}4\\1\\0\end{bmatrix}$ ☐ basis of $\mathbb{R}^3$ ☐ not a basis of $\mathbb{R}^3$

(d) $\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\7\\0\end{bmatrix}, \begin{bmatrix}1\\0\\5\end{bmatrix}$ ☐ basis of $\mathbb{R}^3$ ☐ not a basis of $\mathbb{R}^3$

(e) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}$ ☐ basis of $\mathbb{R}^3$ ☐ not a basis of $\mathbb{R}^3$

Solution.

(a) These vectors are dependent (because of the zero vector) and hence not a basis.

(b) Any basis of $\mathbb{R}^3$ has to have three vectors. Two vectors cannot constitute a basis for $\mathbb{R}^3$.

(c) Any basis of $\mathbb{R}^3$ has to have three vectors. Four vectors cannot constitute a basis for $\mathbb{R}^3$.

(d) These are three vectors, which is the right number for a basis of $\mathbb{R}^3$. Since they are clearly independent (why?!), they form a basis of $\mathbb{R}^3$.

(e) These vectors are dependent (why?!) and hence not a basis. ☐

Problem 2. Decide whether the vectors $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\6\\7\end{bmatrix}$ are a basis of $\mathbb{R}^3$. (Make sure to show your work!)

Solution. These are three vectors, which is the right number for a basis of $\mathbb{R}^3$. They form a basis if and only if they are linearly independent.

$$\begin{bmatrix}1 & -1 & 1\\2 & 2 & 6\\3 & 1 & 7\end{bmatrix} \underset{\rightsquigarrow}{\substack{R_2-2R_1\Rightarrow R_2\\R_3-3R_1\Rightarrow R_3}} \begin{bmatrix}1 & -1 & 1\\0 & 4 & 4\\0 & 4 & 4\end{bmatrix} \underset{\rightsquigarrow}{R_3-R_2\Rightarrow R_3} \begin{bmatrix}1 & -1 & 1\\0 & 4 & 4\\0 & 0 & 0\end{bmatrix}$$

The third column does not contain a pivot, so the system $\begin{bmatrix}1 & -1 & 1\\2 & 2 & 6\\3 & 1 & 7\end{bmatrix}\boldsymbol{x} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ has a free variable and hence non-trivial solutions. This means that the three vectors are linearly dependent. They do not form a basis of $\mathbb{R}^3$. ☐

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