Quiz #3

MATH 237 — Linear Algebra I
Thursday, Sept 10

Please print your name:

Problem 1. Determine if $\begin{bmatrix} -10 \\ -8 \\ 7 \end{bmatrix}$ is in the span of the vectors $\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 8 \\ -7 \end{bmatrix}$.

As always, you must show work to get credit.

Solution. The question is equivalent to asking whether there exist values x_1 and x_2 such that

$$x_1\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ 8 \\ -7 \end{bmatrix} = \begin{bmatrix} -10 \\ -8 \\ 7 \end{bmatrix}.$$

This is the vector form of the linear system with augmented matrix $\left[\begin{array}{cc|c} -1 & 2 & -10 \\ 4 & 8 & -8 \\ -3 & -7 & 7 \end{array}\right]$.

We therefore eliminate!

$$\left[\begin{array}{cc|c} -1 & 2 & -10 \\ 4 & 8 & -8 \\ -3 & -7 & 7 \end{array}\right] \overset{\substack{R_2+4R_1 \Rightarrow R_2 \\ R_3-3R_1 \Rightarrow R_3}}{\rightsquigarrow} \left[\begin{array}{cc|c} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & -13 & 37 \end{array}\right] \overset{R_3+\frac{13}{16}R_2 \Rightarrow R_3}{\rightsquigarrow} \left[\begin{array}{cc|c} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & 0 & -2 \end{array}\right]$$

The last row shows that this system does not have a solution.

Hence, $\begin{bmatrix} -10 \\ -8 \\ 7 \end{bmatrix}$ is not in the span of the vectors $\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 8 \\ -7 \end{bmatrix}$. □

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