Homework #9

MATH 237 — Linear Algebra I
due Thursday, Dec 3, in class

Please print your name:

Problem 1. For each matrix A, find the eigenvalues of A as well as a basis for the corresponding eigenspaces.

(a) $\begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Problem 2. The processors of a supercomputer are inspected weekly in order to determine their condition. The condition of a processor can either be perfect, good, reasonable or bad.

A perfect processor is still perfect after one week with probability 0.7, with probability 0.2 the state is good, and with probability 0.1 it is reasonable. A processor in good conditions is still good after one week with probability 0.6, reasonable with probability 0.2, and bad with probability 0.2. A processor in reasonable condition is still reasonable after one week with probability 0.5 and bad with probability 0.5. A bad processor must be repaired. The reparation takes one week, after which the processor is again in perfect condition.

In the steady state, what is the percentage of processors in perfect condition?

Problem 3. Consider $A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$.

(a) What is the geometric interpretation of the linear map $\boldsymbol{x} \mapsto A\boldsymbol{x}$, which maps $\mathbb{R}^2$ to $\mathbb{R}^2$?

[It helps to make a sketch of where the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are being sent. If still stuck, look again at Example 145.]

(b) What is the geometric interpretation of the linear map $\boldsymbol{x} \mapsto A^2\boldsymbol{x}$? (No computation here.)

(c) Compute A^2 by multiplying A with itself.

[From your interpretations it follows that $A^2 = \begin{bmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{bmatrix}$. By comparing the bottom left entries of this with your computation of A^2, you can conclude that $\sin(2\theta) = 2\cos(\theta)\sin(\theta)$. You proved a trig identity!]

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