Homework #6

MATH 237 — Linear Algebra I
due Thursday, Oct 22, in class

Please print your name:

Problem 1. Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \\ 3 & 6 & 2 & 22 \\ 4 & 8 & 0 & 16 \end{bmatrix}$.

(a) Find a basis for $\mathrm{col}(A)$. What is the dimension of $\mathrm{col}(A)$?

(b) Is the vector $\boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ in $\mathrm{col}(A)$?

Hint: Note that this is the same question as: "Does $A\boldsymbol{x} = \boldsymbol{u}$ have a solution?"
However, save yourself time and observe that if, say, $\boldsymbol{v}_1, \boldsymbol{v}_2$ form a basis for $\mathrm{col}(A)$, then you only need to determine whether the simpler system $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 = \boldsymbol{u}$ has a solution (because we got rid of free variables, this system either has a unique solution or none at all).

(c) Find a basis for $\mathrm{col}(A^T)$. What is the dimension of $\mathrm{col}(A^T)$?

(d) Is the vector $\boldsymbol{w} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix}$ in $\mathrm{col}(A^T)$? Is $\boldsymbol{w}$ in $\mathrm{col}(A)$? Conclude that $\mathrm{col}(A) \neq \mathrm{col}(A^T)$ (but both spaces have the same dimension).

(e) If possible, write the vector $\boldsymbol{a} = \begin{bmatrix} 3 \\ 4 \\ 13 \\ 12 \end{bmatrix}$ as a linear combination of your basis of $\mathrm{col}(A)$.

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