Homework #5

MATH 237 — Linear Algebra I
due Tuesday, Sep 29, in class

Please print your name:

Problem 1. Consider $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) Calculate A^{-1}.

(b) Using (a), solve the system $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

Problem 2. Consider the vectors $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, $\boldsymbol{v}_4 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

(a) Are the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4$ linearly independent? (No computation needed!)

(b) Use part (b) of Problem 1 to write $\boldsymbol{v}_4$ as a linear combination of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$.

(c) Are the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ linearly independent? (Compare with part (a) of Problem 1!)

If no, then write down a non-trivial linear relation of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ which gives $\mathbf{0}$.

(d) Are the vectors $\boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4$ linearly independent?

If no, then write down a non-trivial linear relation of $\boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4$ which gives $\mathbf{0}$.

(e) Using your work in (c), decide whether the following statements are true or false:

The system $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution.	TRUE	FALSE
The system $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is always consistent.	TRUE	FALSE
The system $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ always has a unique solution.	TRUE	FALSE
The matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.	TRUE	FALSE

(f) Using your work in (d), decide whether the following statements are true or false:

The system $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & -1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution.	TRUE	FALSE
The system $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & -1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is always consistent.	TRUE	FALSE
The system $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & -1 \end{bmatrix}\boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ always has a unique solution.	TRUE	FALSE
The matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & -1 \end{bmatrix}$ is invertible.	TRUE	FALSE

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