Homework #4

MATH 237 — Linear Algebra I
due Thursday, Sep 17, in class

Please print your name:

Problem 1. Consider

$$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ -1 & 2 & 1 & 2 \\ 1 & -2 & 3 & 2 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \qquad \boldsymbol{c} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}.$$

(a) Find (in vector form) the general solution to the linear system $A\boldsymbol{x} = \boldsymbol{b}$.

(b) From your answer in (a), deduce the general solution to the associated homogeneous linear system $A\boldsymbol{x} = \boldsymbol{0}$.

[You should not have to do any computations!]

(c) Verify that $\boldsymbol{x} = [\ 0 \ \ 1 \ \ 1 \ \ 1\ ]^T$ is a (particular) solution to the linear system $A\boldsymbol{x} = \boldsymbol{c}$.

[You should not solve the system. Just multiply a matrix with a vector.]

(d) Using the information from (b) and (c), find the general solution to the linear system $A\boldsymbol{x} = \boldsymbol{c}$.

[Again, you should not have to do any computations!]

(e) Compute A^TA. (Make sure that your answer is a symmetric 4×4 matrix.)

(f) **(Bonus)** Suppose that A is any $m \times n$ matrix. Can you give a reason why A^TA is always a symmetric matrix?

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