Preparing for the Final

MATH 237 — Linear Algebra I
Thursday, Dec 10

Please print your name:

Problem 1. Go over all past quizzes!

Problem 2. Study the practice problems for the two midterm exams!

Problem 3. Retake the two midterm exams!

(A copy without solutions is available on our course website. Of course, you also find solutions there.)

Additional problems covering the material since the second midterm

Problem 4. Find the eigenvalues and bases for the eigenspaces of A.

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 1 & 6 \\ 2 & 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Problem 5. Compute the following determinants by expanding by the first column.

Here, i is the imaginary unit. All you need to know about it is that $i^2 = -1$.

(a) $\begin{vmatrix} 1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & i \\ i & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & i & \\ i & 1 & i \\ & i & 1 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & i & & \\ i & 1 & i & \\ & i & 1 & i \\ & & i & 1 \end{vmatrix}$

(e) $\begin{vmatrix} 1 & i & & & \\ i & 1 & i & & \\ & i & 1 & i & \\ & & i & 1 & i \\ & & & i & 1 \end{vmatrix}$

(f) Can you guess what the next determinant will be?

Problem 6. Suppose there is an epidemic in which, every month, half of those who are well become sick, and a quarter of those who are sick become dead. What is the proportion of dead people in the long term equilibrium (steady state).

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Some short answer problems

Problem 7. What are the eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ -1 & 1 & 3 & 0 \\ 0 & 1 & 2 & 4 \end{bmatrix}$?

Problem 8. Write down the cofactor expansion of $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ along

(a) the second row,

(b) the third column.

Problem 9. If A and B are 3×3 matrices with $\det(A) = 4$ and $\det(B) = -1$. What is the determinant of $C = 2A^T A^{-1} B A$?

Problem 10. Let A be a 7×7 matrix with $\dim \operatorname{null}(A) = 1$. What can you say about $\det(A)$?

Problem 11. Let A be a $n \times n$ matrix with $A^T = A^{-1}$. What can you say about $\det(A)$?

Problem 12. Let A be a 5×5 matrix with $\dim \operatorname{row}(A) = 5$. What can you say about $\det(A)$?

Problem 13. What is $\dim \operatorname{null}\left(\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \right)$?

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