Math 415 - Midterm 1

Thursday, September 25, 2014

Circle your section:

Philipp Hieronymi	2pm	3pm
Armin Straub	9am	11am

Name:

NetID:

UIN:

Problem 0. [**1 point**] Write down the number of your discussion section (for instance, AD2 or ADH) and the first name of your TA (Allen, Anton, Babak, Mahmood, Michael, Nathan, Tigran, Travis).

Section:	TA:

To be completed by the grader:

0	1	2	3	4	5	Shorts	$\sum$
/1	/11	/15	/15	/12	/8	/21	/83

Good luck!

Instructions

- No notes, personal aids or calculators are permitted.
- This exam consists of 9 pages. Take a moment to make sure you have all pages.
- You have 75 minutes.
- Answer all questions in the space provided. If you require more space to write your answer, you may continue on the back of the page (make it clear if you do).
- **Explain your work!** Little or no points will be given for a correct answer with no explanation of how you got it.
- In particular, you have to **write down all row operations** for full credit.

Problem 1. Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

(a) [**8 points**] Determine A^{-1}.

(b) [**3 points**] Using A^{-1}, solve $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

Problem 2. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix}.$$

(a) [**10 points**] Calculate the LU decomposition of A.

(b) [**5 points**] Solve

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 7 \end{bmatrix}$$

without reducing the augmented matrix, but using the LU decomposition.

Problem 3. Consider the following system of linear equations:

$$\begin{array}{rrrrcr} x_1 & -2x_2 & +x_3 & & = & 1 \\ -x_1 & +2x_2 & +x_3 & +2x_4 & = & 1 \\ -2x_1 & +4x_2 & +4x_3 & +6x_4 & = & 4 \end{array}$$

(a) [**2 points**] Write down the augmented matrix corresponding to this system.
(b) [**7 points**] Determine the row reduced echelon form of the augmented matrix.
(c) [**6 points**] Use your result in (b) to find a parametric description of the set of solutions to the system of linear equations.

Problem 4. Let

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ h \\ 3h \end{bmatrix}, \quad \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) [**8 points**] For which value of h is $\boldsymbol{w}$ a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$?

(b) [**4 points**] For the value of h found in (a), write down the linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ which gives $\boldsymbol{w}$.

Problem 5. **[8 points]** Determine which of the following sets are a subspace of the vector space of all 2×2 matrices. In each case, give a short reason.

(a) $W_1 = \left\{ \begin{bmatrix} 2a & b \\ b & 3a \end{bmatrix} \ : \ a, b \text{ in } \mathbb{R} \right\}$

(b) $W_2 = \left\{ \begin{bmatrix} 2a & b \\ b & 3a \end{bmatrix} \ : \ a, b \text{ in } \mathbb{R} \text{ and } a + b = 1 \right\}$

SHORT ANSWERS

[21 points overall, 3 points each]

Instructions: The following problems have a short answer. No reason needs to be given. If the problem is multiple choice, circle the correct answer (there is always exactly one correct answer).

Short Problem 1. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$. Compute $A^T A$.

$A^T A =$

Short Problem 2. Let A be a matrix such that, for every $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$, $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2y \\ x \\ x - z \end{bmatrix}$.

Then, what is A?

$A =$

Short Problem 3. Let C be a 3×4 matrix such that C has two pivot columns, and let $\boldsymbol{d}$ be a vector in $\mathbb{R}^3$. Is it true that, if the equation $C\boldsymbol{x} = \boldsymbol{d}$ has a solution, then it has infinitely many solutions?

(a) True.
(b) False.
(c) Unable to determine.

Short Problem 4. Let

$$A = \begin{bmatrix} a & a+1 \\ a+1 & a \end{bmatrix}.$$

For which choice(s) of a is the matrix A *not* invertible?

Short Problem 5. There is one vector which every subspace of $\mathbb{R}^2$ has to contain. Which vector is that?

Short Problem 6. Let W_1 be the set of all polynomials $p(t)$ which have a zero at $t = 1$ (that is, $p(1) = 0$), and let W_0 be the set of all polynomials $p(t)$ which have a zero at $t = 0$. Are these sets subspaces of the vector space of all polynomials?

(a) Both W_0 and W_1 are subspaces.
(b) Only W_0 is a subspace.
(c) Only W_1 is a subspace.
(d) Neither W_0 nor W_1 are subspaces.

Short Problem 7. Let $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$. Which of the following is true?

(a) W is empty.
(b) W is a line.
(c) W is a plane.
(d) W is all of $\mathbb{R}^3$.