Math 415 - Midterm 1

Thursday, September 25, 2014

Circle your section:

Philipp Hieronymi	2pm	3pm
Armin Straub	9am	11am

Name:

NetID:

UIN:

Problem 0. [1 point] *Write down the number of your discussion section (for instance, AD2 or ADH) and the first name of your TA (Allen, Anton, Babak, Mahmood, Michael, Nathan, Tigran, Travis).*

Section:	*TA:*

To be completed by the grader:

0	1	2	3	4	5	6	Shorts	$\sum$
/1	/10	/15	/15	/10	/10	/12	/21	/94

Good luck!

Instructions

- No notes, personal aids or calculators are permitted.
- This exam consists of 9 pages. Take a moment to make sure you have all pages.
- You have 75 minutes.
- Answer all questions in the space provided. If you require more space to write your answer, you may continue on the back of the page (make it clear if you do).
- **Explain your work!** Little or no points will be given for a correct answer with no explanation of how you got it.
- In particular, you have to **write down all row operations** for full credit.

Problem 1. *Let*

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

(a) **[8 points]** *Determine* A^{-1}.
(b) **[2 points]** *Check whether* $AA^{-1} = I_3$.

Problem 2. *Consider the matrix*

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 3 & 1 & 2 \\ 6 & 7 & 5 \end{bmatrix}.$$

(a) **[10 points]** *Calculate the LU decomposition of A.*

(b) **[5 points]** *Solve*

$$\begin{bmatrix} 3 & 2 & 0 \\ 3 & 1 & 2 \\ 6 & 7 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$$

without reducing the augmented matrix, but using the LU decomposition.

Problem 3. *Let*

$$B = \begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix}.$$

(a) **[8 points]** *Determine the reduced echelon form of B.*

(b) **[7 points]** *Use your result in* (a) *to find a parametic description of the set of solutions of the following system of linear equations:*

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

Problem 4. **[10 points]** *Consider the vectors*

$$\boldsymbol{w} = \begin{bmatrix} 2 \\ -4 \\ -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Is $\boldsymbol{w}$ *in* $\operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$*? Show your calculations!*

Problem 5. [10 points] *Let*

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ h_1 \\ h_2 \end{bmatrix}.$$

For which values of h_1 *and* h_2 *is* $\boldsymbol{v}_3$ *a linear combination of* $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$*?*

Problem 6. **[12 points]** *Determine which of the following sets are subspaces and* give reasons*:*

(a) $W_1 = \{\begin{bmatrix} a \\ b \end{bmatrix} \ : \ ab = 0\}$,

(b) $W_2 = \{\begin{bmatrix} a+1 \\ a \end{bmatrix} \ : \ a \ in \ \mathbb{R}\}$,

(c) $W_3 = \{\begin{bmatrix} a \\ b \end{bmatrix} \ : \ a^2 + b^2 \leq 1\}$.

SHORT ANSWERS

[21 points overall, 3 points each]

Instructions: The following problems have a short answer. No reason needs to be given. If the problem is multiple choice, circle the correct answer (there is always exactly one correct answer).

Short Problem 1. *Let A be a matrix such that, for every $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$, $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y+z \\ 0 \\ 2x-z \end{bmatrix}$.*

Then, what is A?

$A =$

Short Problem 2. *Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 3 \end{bmatrix}$. Then, what is A^T?*

$A^T =$

Short Problem 3. *The set of solutions in $\mathbb{R}^3$ of the equation*

$$x_1 - 3x_2 + 2x_3 = 1$$

is

(a) *empty,*
(b) *a line not through the origin,*
(c) *a line through the origin,*
(d) *a plane.*

Short Problem 4. *Let A be an $l \times m$ matrix and B be an $n \times p$ matrix. Under which condition is $A^T B$ defined?*

Short Problem 5. *Let C be a 3×4 matrix such that C has two pivot columns. Is it true that the equation $C\boldsymbol{x} = \boldsymbol{d}$ has a solution for every $\boldsymbol{d}$ in $\mathbb{R}^3$.*

(a) *True.*
(b) *False.*
(c) *Unable to determine.*

Short Problem 6. *Let*

$$A = \begin{bmatrix} 3 & a-6 \\ 3a & -a+6 \end{bmatrix}.$$

For which choices of a is the matrix A invertible?

Short Problem 7. *How many solutions has a linear system with 4 equations and 5 unknowns?*

(a) *The system either has no solution or infinitely many solutions.*
(b) *The system has no solution.*
(c) *The system has exactly one solution.*
(d) *The system has infinitely many solution.*