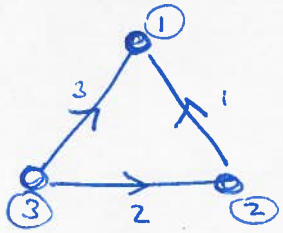
1

(1)



(2) from matrix:

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \overset{\text{RREF}}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

parametric solution of $A\vec{x} = \vec{0}$

$\vec{x} = \begin{pmatrix} x_3 \\ x_3 \\ x_3 \end{pmatrix}$ with x_3 free

$\Rightarrow$ $\mathrm{Nul}(A)$ has basis: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

from graph:

the graph has only one connected subgraph

$\Rightarrow$ $\mathrm{Nul}(A)$ has basis $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(3) from matrix:

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix} \overset{\text{RREF}}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ again, x_3 only free variable

$\mathrm{Nul}(A^T)$ has basis: $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

(3) continued

from graph:

there is only one loop:

edge_1, $-\text{edge}_3$, edge_2

$\Rightarrow$ $\text{Nul}(A^T)$ has basis: $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

(4)

$\vec{b}$ is in $\text{Col}(A)$

$\overset{FT}{\iff}$ $\vec{b}$ is orthogonal to $\text{Nul}(A^T)$

$\iff$ $\vec{b} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = b_1 + b_2 - b_3 = 0$

(Means sum of potential differences around a loop is zero — Kirchhoff's voltage law)

(5)

$\vec{f}$ is in $\text{Col}(A^T)$

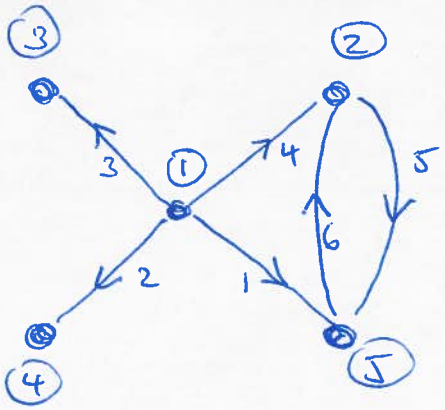
$\overset{FT}{\iff}$ $\vec{f}$ is orthogonal to $\text{Nul}(A)$

$\iff$ $\vec{f} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = f_1 + f_2 + f_3 = 0$

(Means sum of net currents over connected subgraphs is zero)

2

(1)



(2) The graph is connected

$\Rightarrow$ basis for $Nul(A)$: $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

(3) There are two (independent) loops:

* $edge_1, edge_6, -edge_4$
* $edge_5, edge_6$

$\Rightarrow$ basis for $Nul(A^T)$: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

(4) not relevant for exam