Preparation problems for the discussion sections on September 23rd and 25th

1. *Determine which of the following sets are subspaces and* give reasons*:*

(a) $W_1 = \{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \ : \ a - 2b = c, 4a + 2c = 1 \}$,

(b) $W_2 = \{ \begin{bmatrix} a - b \\ c \\ a + c \\ a - 2b - c \end{bmatrix} \ : \ a, b, c \in \mathbb{R} \}$,

(c) $W_3 = \{ \begin{bmatrix} a \\ b \end{bmatrix} \ : \ a \cdot b \geq 0 \}$,

(d) $W_4 = \{ \begin{bmatrix} a \\ b \end{bmatrix} \ : \ a^2 + b^2 \leq 1 \}$.

Also draw the sets W_3 and W_4 and give geometric reasons why W_3 and W_4 are not subspaces.

2. *Is* $H = \left\{ \begin{bmatrix} a + 1 \\ a \end{bmatrix} : \ a \text{ in } \mathbb{R} \right\}$ *a subspace of* $\mathbb{R}^2$*? Why or why not?*

Is $K = \left\{ \begin{bmatrix} a + 1 \\ b \end{bmatrix} : \ a \text{ and } b \text{ in } \mathbb{R} \right\}$ *a subspace of* $\mathbb{R}^2$*? Why or why not?*

3. *Is the set H of all matrices of the form* $\begin{bmatrix} 2a & b \\ 3a + b & 3b \end{bmatrix}$ *a subspace of $M_{2\times 2}$? Explain.*

4. *A matrix B is called symmetric if $B^T = B$. Let V be the set of all symmetric 2×2-matrices. Is V a subspace of $M_{2\times 2}$?*