Inhomogeneous heat equation

EG Solve: $u_t = 3u_{xx} + 4x^2$ (inhomogeneous) — PDE

$u(0,t) = 1 \quad u_x(3,t) = -5$ — BC

$u(x,0) = f(x)$ — IC

main idea $u(x,t) = v(x) + w(x,t)$

$v(x)$: steady-state solution

$w(x,t)$: transient solution, $w(x,t) \to 0$ as $t \to \infty$ (same for derivatives)

PDE $w_t = 3v'' + 3w_{xx} + 4x^2$ ($w_t \to 0$, $w_{xx} \to 0$)

let $t \to \infty$: $0 = 3v'' + 4x^2$

thus: $w_t = 3w_{xx}$

BC $v(0) + w(0,t) = 1 \qquad v'(3) + w_x(3,t) = -5$

let $t \to \infty$: $v(0) = 1 \qquad v'(3) = -5$

thus: $w(0,t) = 0 \qquad w_x(3,t) = 0$

steady-state solution $v(x)$

$3v'' + 4x^2 = 0$

$v(0) = 1 \quad v'(3) = -5$

solve → $v(x) = -\frac{1}{9}x^4 + 7x + 1$

transient solution $w(x,t)$

$u(x,0) = f(x)$

$v(x) + w(x,0)$

$w_t = 3w_{xx}$ — PDE

$w(0,t) = 0 \quad w_x(3,t) = 0$ — BC

$w(x,0) = f(x) - v(x)$ — IC

homogeneous heat eq.!

(know how to solve)