Solving systems of DEs

recurrences	differential equations
$a_{n+1} = M a_n$	$y' = M y$

solution corresponding to λ-eigenvector v of M

$a_n = \lambda^n \cdot v$ $\qquad\qquad$ $y(x) = e^{\lambda x} \cdot v$

fundamental matrix solution

[columns are ordinary solutions]

$\Phi_{n+1} = M \Phi_n$ $\qquad\qquad$ $\Phi' = M \Phi$

special fundamental matrix solution

M^n $\qquad\qquad$ e^{Mx} matrix exponential!

(the unique Φ_n with $\Phi_0 = I$) $\qquad$ (the unique $\Phi(x)$ with $\Phi(0) = I$)

can be obtained from any other fundamental matrix solution:

$M^n = \Phi_n \Phi_0^{-1}$ $\qquad\qquad$ $e^{Mx} = \Phi(x)\, \Phi(0)^{-1}$

unique solution to initial value problem

$a_{n+1} = M a_n \quad a_0 = c$ $\qquad\qquad$ $y' = M y \quad y(0) = c$

$\Longrightarrow$ $a_n = M^n c$ $\qquad\qquad$ $\Longrightarrow$ $y(x) = e^{Mx} c$