Inhomogeneous LDEs with constant coefficients

part 2

THM (recipe) $p(D)y = f(x)$ How to find y_p:

① Find $q(D)$ so that $q(D)f(x) = 0$.

② $p(D)$: roots $r_1, r_2, \ldots, r_n$ "old roots"

$q(D)$: roots $s_1, s_2, \ldots, s_m$ "new roots"

③ Then y_p solves $q(D)\,p(D)\,y = 0$.

Let $v_1, v_2, \ldots, v_m$ be the "new" solutions.
(i.e. not solutions to $p(D)y = 0$)

$\Longrightarrow y_p = C_1 v_1 + C_2 v_2 + \ldots + C_m v_m$
for unique values of C_i (found by plugging into DE)

EG $y'' + 4y' + 4y = 7e^{-2x}$

"old" roots: $-2, -2$ "old" solutions: e^{-2x}, xe^{-2x}

"new" root: -2 "new" solution: $x^2 e^{-2x}$

$\Longrightarrow y_p = C x^2 e^{-2x}$

find C by plugging into DE:

$y_p' = C(-2x^2 + 2x)e^{-2x}$ $y_p'' = C(4x^2 - 8x + 2)e^{-2x}$

$y_p'' + 4y_p' + 4y_p = 2Ce^{-2x} \overset{!}{=} 7e^{-2x}$

$\Longrightarrow C = \frac{7}{2}$

general solution:

$\frac{7}{2}x^2 e^{-2x} + C_1 e^{-2x} + C_2 x e^{-2x}$

$= \left(C_1 + C_2 x + \frac{7}{2}x^2\right)e^{-2x}$