Midterm #1

- No notes, personal aids or calculators are permitted.

- Answer all questions in the space provided. If you require more space to write your answer, you may continue on the back of the page. There is a blank page at the end of the exam for rough work.

- **Explain your work!** Little or no points will be given for a correct answer with no explanation of how you got it.

Good luck!

Problem 1. (5 points) Circle the slope field below which belongs to the differential equation $e^x y' = x - y$.
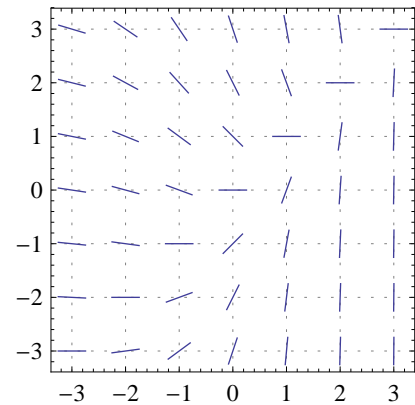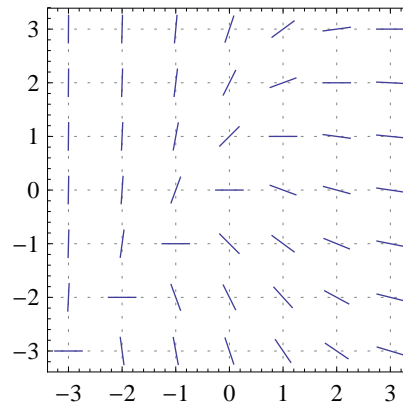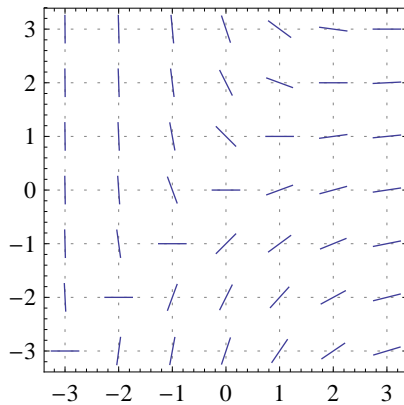
Problem 2. (20 points) Find the general solution to the differential equation $y^{(5)} - 4y^{(4)} + 4y^{(3)} = 0$.

$y(x) =$

Problem 3. (20 points) Solve the initial value problem

$$(x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x} + xy = \frac{1}{\sqrt{x^2+1}}, \qquad y(0)=1.$$

$y(x) =$

Problem 4. (20 points) The time rate of change of a rabbit population P is proportional to the square root of P. At time $t=0$, the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be after two months?

$P(2 \text{ months}) =$

Problem 5. (20 points) For each $c \geqslant 0$, let $y_c(x) = \begin{cases} x^3, & \text{if } x < 0, \\ 0, & \text{if } 0 \leqslant x \leqslant c, \\ (x-c)^3, & \text{if } x > c. \end{cases}$

(a) Sketch the graph of $y_c(x)$ for $c = 3/2$.

(b) Show that, for all $c \geqslant 0$, y_c is a solution to the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3y^{2/3}, \quad y(0) = 0.$$

(c) Explain why (b) does not contradict the theorem on existence and uniqueness for solutions to initial value problems.

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Problem 6. (20 points) Find a general solution to the differential equation

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} - x^2 - y^2 - 3xy = 0.$$

$y(x) =$

Problem 7. (5 points) Consider the differential equation   *Hint:* Do not attempt to solve the DE.

$$y' = y^4 + x^4 + 1.$$

Is it possible that there exists a solution with the property that $\lim_{x \to \infty} y(x) = -\infty$? Why, or why not?

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