Final: practice

MATH 238 — Differential Equations I
final exam: Wednesday, Dec 10, 2025

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

(a) Do the practice problems for both midterms.

(b) Retake both midterm exams.

(c) Do the problems below. (Solutions are posted.)

Problem 2.

(a) Determine the Laplace transform $\mathcal{L}(2e^{4t} - 6e^{-t} + 3)$.

(b) Determine the Laplace transform $\mathcal{L}((2t-5)e^{-3t})$.

(c) Given $f(t) = \begin{cases} 2e^{3t}, & \text{if } t \geqslant \pi, \\ 0, & \text{otherwise,} \end{cases}$ determine its Laplace transform $\mathcal{L}(f(t))$.

(d) Determine the inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{s-16}{s^2-2s-8}\right)$.

Problem 3.

(a) Determine the Laplace transform $5\cosh(3t)$.

(b) Write down a homogeneous linear differential equation satisfied by $y(x) = (5+2x)e^x - 4x^2\cosh(7x)$.

Problem 4. Determine the Laplace transform of the unique solutions to the following initial value problems.

[Do not determine the solution and do not simplify.]

(a) $y'' + 4y' - 3y = 2e^{-4t} + 5t^2,\ y(0) = 7,\ y'(0) = -2$.

(b) $y'' - 6y' + 5y = \begin{cases} 3, & \text{if } 1 \leqslant t < 4, \\ 0, & \text{otherwise,} \end{cases},\ y(0) = 2,\ y'(0) = -1$.

(c) $y_1' = 3y_1 - 4y_2 - 5t^2e^{-3t},\ y_2' = y_1 + 2y_2,\ y_1(0) = 3,\ y_2(0) = -2$.

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For the final exam, you will be provided the following table of Laplace transforms.

$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - s f(0) - f'(0)$
e^{at}	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{at}f(t)$	$F(s-a)$
$t f(t)$	$-F'(s)$
$u_a(t)f(t-a)$	$e^{-as}F(s)$

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