Quiz #10

MATH 126 — Calculus II
Friday, Nov 20

Please print your name:

Problem 1.

(a) When does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge? Make sure to indicate a reason!

(b) When does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge absolutely? Make sure to indicate a reason!

Solution.

(a) If $p > 0$, then the series converges by the alternating series test, because $a_n = \frac{1}{n^p}$ is positive, decreasing, and $\lim_{n\to\infty} a_n = 0$. If $p \leqslant 0$, then $\lim_{n\to\infty} \frac{(-1)^n}{n^p}$ is not zero. Therefore, the series diverges.
In summary, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges if and only if $p > 0$.

(b) By definition, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges absolutely if and only if $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^p}\right| = \sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.
Since this is just the usual p-series, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges absolutely if and only if $p > 1$. □

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