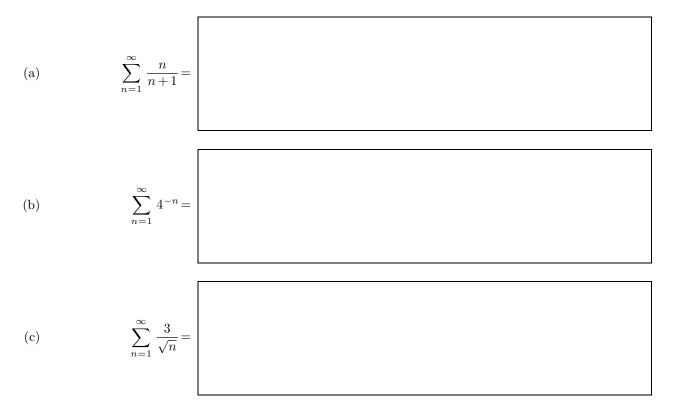
## Please print your name:

Quiz #8

Problem 1. Determine whether the following series converge or diverge. If they converge, determine their value.

If a series diverges, make sure to indicate why!



Solution.

(a) 
$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$
 diverges because  $\frac{n}{n+1} \to 1 \neq 0$  as  $n \to \infty$ .  
(b)  $\sum_{n=1}^{\infty} 4^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n - 1 = \frac{1}{1 - \frac{1}{4}} - 1 = \frac{1}{3}$   
(c)  $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$  diverges because it is a *p*-series with  $p = \frac{1}{2} \leq 1$ .

**Problem 2.** Use the integral test to determine whether  $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$  converges or diverges.

**Solution.** 
$$\sum_{n=1}^{\infty} \frac{n}{n^2+4}$$
 converges if and only if  $\int_{1}^{\infty} \frac{x}{x^2+4} dx$  converges.

First, however, we should verify that the integral test indeed applies: the function  $\frac{x}{x^2+4}$  is obviously positive and continuous for  $x \ge 1$ . For x > 2, it is also decreasing, because  $\frac{x^2+4}{x} = x + \frac{4}{x}$  is increasing (its derivative is  $1 - \frac{4}{x^2}$ , which is positive if x > 2).

Substituting  $u = x^2 + 4$ , we find

$$\int_{1}^{\infty} \frac{x}{x^{2}+4} \, \mathrm{d}x = \frac{1}{2} \int_{5}^{\infty} \frac{\mathrm{d}u}{u} = [\ln |u|]_{5}^{\infty} = \infty$$

because  $\lim_{u \to \infty} \ln |u| = \infty$ . Hence  $\int_1^\infty \frac{x}{x^2 + 4} \, dx$  diverges, and we conclude that  $\sum_{n=1}^\infty \frac{n}{n^2 + 4}$  diverges.  $\Box$