Quiz #7

MATH 126 — Calculus II
Friday, Oct 23

Please print your name:

Problem 1. Determine whether the following series converge or diverge. If they converge, determine their value.

No need to simplify any values!

(a) $$\sum_{n=2}^{\infty} 2^n =$$

(b) $$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\log n} =$$

(c) $$\sum_{n=2}^{\infty} 2^{-n} =$$

(d) $$\sum_{n=0}^{\infty} \frac{2^n + 3^n}{5^n} =$$

Solution.

(a) $\displaystyle\sum_{n=2}^{\infty} 2^n$ diverges because $2^n \to \infty$ as $n \to \infty$.

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\log n}$ diverges because $\frac{\sqrt{n}}{\log n} \to \infty$ as $n \to \infty$.

(c) $$\sum_{n=2}^{\infty} 2^{-n} = \sum_{n=0}^{\infty} 2^{-n} - 1 - \frac{1}{2} = \frac{1}{1-\frac{1}{2}} - 1 - \frac{1}{2} = \frac{1}{2}$$

(d) $$\sum_{n=0}^{\infty} \frac{2^n + 3^n}{5^n} = \sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n + \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{1}{1-\frac{2}{5}} + \frac{1}{1-\frac{3}{5}} = \frac{5}{3} + \frac{5}{2} = \frac{25}{6}$$ □

Problem 2. Express $0.\bar{7} = 0.7777...$ as a quotient of two integers.

Solution. $$0.\bar{7} = 0.7777... = \frac{7}{10} + \frac{7}{10^2} + \frac{7}{10^3} + ... = \frac{7}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + ...\right) = \frac{7}{10} \frac{1}{1-\frac{1}{10}} = \frac{7}{9}$$ □

Problem 3. For which values of x does $\displaystyle\sum_{n=0}^{\infty} 2^n x^n$ converge? Evaluate the series (as a function of x) for these values.

Solution. $\displaystyle\sum_{n=0}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} (2x)^n = \frac{1}{1-2x}$ provided that $|2x| < 1$ (or, equivalently, $|x| < 1/2$). □

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