Midterm #1

Please print your name:

Problem 1. Evaluate the following indefinite integrals.

(a) $\displaystyle \int \frac{\mathrm{d}x}{x} =$

(b) $\displaystyle \int \sin(3x)\,\mathrm{d}x =$

(c) $\displaystyle \int \frac{\mathrm{d}x}{x^2+1} =$
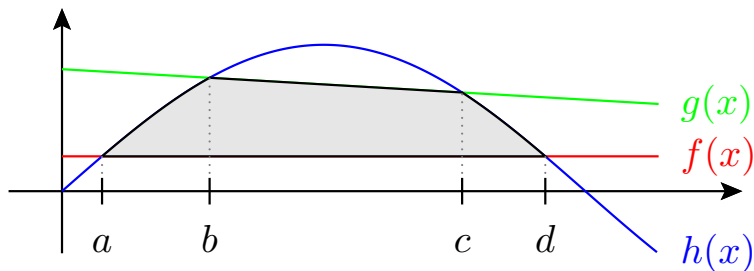
Problem 2. Determine the shape (but not the exact numbers involved) of the partial fraction decomposition of the following rational functions.

(a) $\displaystyle \frac{x}{x^2-1} =$

(b) $\displaystyle \frac{2x+1}{(x^2+1)^2(x+2)} =$

Problem 3. Consider the plot below. What is the area enclosed by the curves $y=f(x)$, $y=g(x)$ and $y=h(x)$? Your answer should be a sum of certain integrals.

Problem 4. Set up an integral (but do not evaluate) for the length of the curve $y=x^3$ for $1 \leqslant x \leqslant 2$.

Problem 5. Evaluate the integral $\int_1^2 x \ln(x)\, dx$.

Problem 6. Evaluate the integral $\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\, dx$.

Problem 7. Solve the initial value problem $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y^2,\ \ y(0) = 1.$

Problem 8. Set up an integral (but do not evaluate) for the volume of the solid obtained by revolving the region enclosed by the curves

$$y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 3,$$

about the line $y = -2$.

Problem 9. (Bonus!) Roughly, what is the speed of light (in vacuum)?