Preparing for Midterm #1

Please print your name:

Problem 1. Go over all the quizzes!

To help you with that, there is a version of each quiz posted on our course website without solutions (of course, there's solutions, too).

Problem 2. Find the length of the following curve:

$$y = 1 - 2x^{3/2}, \quad 0 \leqslant x \leqslant \frac{1}{3}.$$

Problem 3. Evaluate $\displaystyle\int_0^2 \frac{x}{\sqrt{4-x^2}}\, \mathrm{d}x$.

Problem 4. Evaluate $\displaystyle\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, \mathrm{d}x$.

Problem 5. Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{x^2+1}, \quad y(0) = 2.$$

Problem 6. Evaluate $\displaystyle\int_{-2}^2 \frac{1}{x+1}\, \mathrm{d}x$ or show that the integral diverges.

Problem 7. Consider the region enclosed by the curves

$$y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2.$$

(a) Sketch the region and find its area.

(b) Find the volume of the solid obtained by revolving this region about the line $y = 0$.

(c) Find the volume of the solid obtained by revolving this region about the line $y = -1$.

Problem 8. Evaluate $\displaystyle\int x^3 \cos(x^2+1)\, \mathrm{d}x$.

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Problem 9. Consider the region bounded by the curves

$$y = 0, \quad y = \sin(x), \quad 0 \leqslant x \leqslant \pi.$$

Sketch the region, then set up an integral for the volume of the solid obtained by rotating this region about the x-axis. Evaluate this integral using integration by parts.

Problem 10. Consider the cylindrical container displayed to the right. It is half filled with sand weighing 100 lb/ft^3.

(a) Determine the amount of work needed to lift the sand to the rim of the tank.

(b) Determine the amount of work needed to lift the sand to a level 10 ft above the rim of the tank. Just an integral is good enough, here.

(c) Now, suppose the container is completely filled with sand. Determine the amount of work needed to lift the sand to a level 10 ft above the rim of the tank. Again, an integral is good enough, here.

16ft

8ft

sand

10ft