Example 107. Determine the shape (but not the exact numbers involved) of the partial fraction decomposition of the following rational functions.

(a) $\dfrac{x^2 + 5}{x^3 - 2x^2} =$

(b) $\dfrac{x^2 + 5}{(x^3 - 2x^2)^2} =$

(c) $\dfrac{x^2 + 5}{(x^4 + 2x^2)^2} =$

Example 108. Determine the partial fraction decomposition of $\dfrac{x^2 + 5}{x^3 - 2x^2}$.

Your final answer should be $\dfrac{-5/4}{x} + \dfrac{-5/2}{x^2} + \dfrac{9/4}{x - 2}$.

Example 109. Evaluate the following integrals:

(a) $\displaystyle\int_0^2 (x + 1)e^{2x}\,\mathrm{d}x =$

Your final answer should be $\dfrac{5}{4}e^4 - \dfrac{1}{4}$.

(b) $\displaystyle\int_0^2 x\sin\left(\pi x^2\right)\mathrm{d}x =$

Your final answer should be 0.

(c) $\displaystyle\int_0^\infty x e^{-3x}\,\mathrm{d}x =$

Your final answer should be $\dfrac{1}{9}$.