What we have learned so far

In this first half of the semester, everything we did was centered around the integral. Basically, we have seen several techniques to evaluate integrals (indefinite ones [i.e. no limits], definite ones [i.e. with limits] and **improper** ones) and we have seen several applications of integration.

Techniques of integration

- **Substitution**
- **Integration by parts**
- **Partial fractions**

Applications of integration

- We have computed **areas**, **volumes**, **arc lengths** and **physical work**.

  In each case, the basic principle was the same: to calculate something complicated, we broke it into many small pieces (which are easy to calculate) and then summed over all these pieces. The "summing" is precisely what turns into an integral.

- We have seen one application of a different kind, namely **differential equations**.

  We have learned about **separation of variables**, which is a technique to solve certain differential equations (and initial value problems). Once the variables are separated, we have to integrate both sides...

Example 105. Solve the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 \sin(x)$, $y\left(\frac{\pi}{2}\right) = 3$.

Solution. Your final answer should be $y(x) = \frac{1}{\cos(x) + \frac{1}{3}}$.

Example 106. Evaluate the following integrals:

(a) $\int x e^x \, \mathrm{d}x =$

(b) $\int x e^{x^2} \, \mathrm{d}x =$

(c) $\int x \ln(x) \, \mathrm{d}x =$

The best strategy for this integral is integration by parts. However, just for practice, substitute $u = \ln(x)$. The result should be $\int u e^{2u} \, \mathrm{d}u$, which is just like what we have for the first of the three integrals.

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