**Example 68.** Find the general solution to $\frac{dy}{dx} = (1+y)e^x$, $y > -1$, using separation of variables.

**Solution.** $y(x) = e^{e^x + C} - 1$

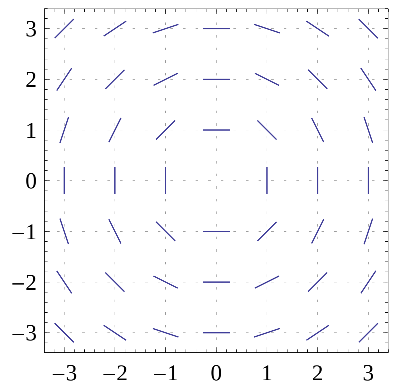
As an exercise in differentiation, verify that $y(x)$ indeed solves the differential equation.

The next example illustrates that we can "plot" solutions to differential equations (it does not matter if we are able to actually solve them).

[This is an important point because "plotting" really means that we can numerically approximate solutions. For complicated systems of differential equations, such as those used to model fluid flow, this is usually the best we can do. Nobody can actually solve these equations.]

**Example 69.** Consider the DE $\frac{dy}{dx} = -x/y$.

Let's pick a point, say, $(1, 2)$. If a solution $y(x)$ is passing through that point, then its slope has to be $\frac{dy}{dx} = -1/2$. We therefore draw a small line through the point $(1, 2)$ with slope $-1/2$. Continuing in this fashion for several other points, we obtain the **slope field** on the right.




With just a little bit of imagination, we can now anticipate the solutions to look like (half)circles around the origin. Let us check whether $y(x) = \sqrt{r^2 - x^2}$ might indeed be a solution!

$y'(x) = \frac{1}{2}\frac{-2x}{\sqrt{r^2 - x^2}} = -x/y(x)$. So, yes, we actually found solutions!

**Example 70.** Find the general solution to $\frac{dy}{dx} = -x/y$ using separation of variables.

**Example 71.** Solve the initial value problem $\frac{dy}{dx} = -x/y$, $y(0) = 2$.

**Solution.** $y(x) = \sqrt{4 - x^2}$

Which differential equations can we actually solve using separation of variables?

- A general DE of first order is typically of the form $\frac{dy}{dx} = f(x, y)$.

  For instance, $\frac{dy}{dx} = \sin(xy) - x^2 y$.
  [First order means that only the first derivative of $y$ shows up. The most general form of a DE of first order is $F(x, y, y') = 0$ but we can usually solve for $y'$ to get to the above form.]

- The ones we can solve are **separable equations**, which are of the form $\frac{dy}{dx} = g(x)h(y)$.

**Example 72.** The equation $\frac{dy}{dx} = y - x$ (although simple) is not separable.

**Example 73.** The equation $\frac{dy}{dx} = e^{y-x}$ is separable because we can write it as $\frac{dy}{dx} = e^y e^{-x}$.

**Example 74.** Solve the initial value problem $\frac{dy}{dx} = xy$, $y(0) = 3$.

**Solution.** $y(x) = 3e^{x^2/2}$

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