Substitution, continued

Example 20. $\int x\sin(x^2+3)\,\mathrm{d}x =$

Example 21. $\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)\mathrm{d}x =$

Review 22. $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$

To verify, use $\ln|x| = \begin{cases} \ln(x), & \text{if } x>0, \\ \ln(-x), & \text{if } x<0, \end{cases}$ to differentiate the right-hand side.

Example 23. $\int \frac{\mathrm{d}x}{x\ln(x)} =$

In class, we used the substitution $u = \ln(x)$.
For practice, use $u = \frac{1}{\ln(x)}$ and see if you can get the same final answer.

(For more complicated integrals, finding the "best" substitution is quite an art form. For the integrals we are concerned with, there is always a natural choice.)

Example 24. $\int_2^4 \frac{\mathrm{d}x}{x\ln(x)} =$

To reduce work, make sure to use the previous problem and the fundamental theorem.

Example 25. $\int 3x^5\sqrt{x^3+1}\,\mathrm{d}x =$

In class, we used the substitution $u = x^3+1$ (that's the most natural choice).
For practice, use $u = \sqrt{x^3+1}$ and see if you can get the same final answer.

Example 26. $\int_0^\pi \frac{\sin(t)}{2-\cos(t)}\mathrm{d}t =$

Here, we want to substitute $u = 2-\cos(t)$.

(Note how we can already anticipate that the derivative will nicely take care of the $\sin(t)$ in the integrand.)

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