Homework #13

MATH 126 — Calculus II
Friday, Dec 4

Please print your name:

Problem 1. (9.9.22) Find the Taylor series at $x=0$ for $\frac{2}{(1-x)^3}$.

Solution. Note that $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\frac{1}{1-x} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{(1-x)^2} = \frac{2}{(1-x)^3}$. Hence, if $|x|<1$, we have

$$\frac{2}{(1-x)^3} = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\frac{1}{1-x} = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\sum_{n=0}^{\infty} x^n = \frac{\mathrm{d}}{\mathrm{d}x}\sum_{n=1}^{\infty} n x^{n-1} = \sum_{n=2}^{\infty} n(n-1)x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1)x^n.$$

[In the last step, we replaced n with $n+2$ in the sum to put it in the usual form of a Taylor series.] □

Problem 2. (9.10.68) Use Euler's identity to show that

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \quad \text{and} \quad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}.$$

Solution. Replacing θ with $-\theta$ in $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ results in $e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos(\theta) - i\sin(\theta)$. Therefore,

$$\frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{[\cos(\theta)+i\sin(\theta)] + [\cos(\theta)-i\sin(\theta)]}{2} = \frac{2\cos\theta}{2} = \cos\theta$$

and, likewise,

$$\frac{e^{i\theta}-e^{-i\theta}}{2i} = \frac{[\cos(\theta)+i\sin(\theta)] - [\cos(\theta)-i\sin(\theta)]}{2i} = \frac{2i\sin\theta}{2i} = \sin\theta.$$

□

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