Quiz #8

MATH 125 — Calculus 1
Tuesday, April 2

Please print your name:

Problem 1. (4 points) Compute $\lim_{x\to 0} \frac{1-\cos(3x)}{x^2}$. [Show your work!]

Solution. $\lim_{x\to 0} \frac{1-\cos(3x)}{x^2} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{LH}}{=}} \lim_{x\to 0} \frac{3\sin(3x)}{2x} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{LH}}{=}} \lim_{x\to 0} \frac{9\cos(3x)}{2} = \frac{9}{2}$ □

Problem 2. (6 points) An open box is folded from a 8 in by 3 in piece of cardboard by cutting congruent squares from the corners and bending up the sides. How large should the cutout squares be to make the box hold as much as possible? [Show your work!]

Solution. Let x be the side length of the squares cut from the corners. Then the volume of the box is

$$V = x(8-2x)(3-2x) = 4x^3 - 22x^2 + 24x.$$

Note that x can range from $x=0$ (zero volume) to $x = \frac{3}{2}$ (zero volume, again). We want to find the absolute maximum of V for x in $\left[0, \frac{3}{2}\right]$.

Since $\frac{\mathrm{d}V}{\mathrm{d}x} = 12x^2 - 44x + 24 = 4(3x^2 - 11x + 6)$, solving $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ yields $x = \frac{11 \pm \sqrt{11^2 - 4\cdot 18}}{6} = \frac{11\pm 7}{6}$ for the critical points. Since $\frac{11+7}{6} = 3 > \frac{3}{2}$, the only critical point in $\left[0, \frac{3}{2}\right]$ is $x = \frac{11-7}{6} = \frac{2}{3}$.

Since $V = 0$ at the endpoints, the absolute max of V over $\left[0, \frac{3}{2}\right]$ must occur at a critical point, meaning the absolute max must be at $x = \frac{2}{3}$.

In conclusion, for a maximum volume box, the cutout squares should have side length $\frac{2}{3} \approx 0.667$ in. □

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