Quiz #6

MATH 125 — Calculus 1
Tuesday, Feb 26

Please print your name:

Problem 1. (4 points) Compute the following derivatives. [No need to show work.]

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln(x^3+7) = \boxed{}$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}[x^4\tan^{-1}(x)] = \boxed{}$

Solution.

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln(x^3+7) = \dfrac{3x^2}{x^3+7}$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}[x^4\tan^{-1}(x)] = 4x^3\tan^{-1}(x) + \dfrac{x^4}{x^2+1}$ □

Problem 2. (6 points) Consider the curve $x^3+2y^3 = xy$. [Show your work!]

(a) Using implicit differentiation, determine $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Determine the line tangent to the curve at the point $(1,-1)$.

Solution.

(a) Applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides of $x^3+2y^3 = xy$, we obtain $3x^2+6y^2\frac{\mathrm{d}y}{\mathrm{d}x} = y + x\frac{\mathrm{d}y}{\mathrm{d}x}$, so that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y-3x^2}{6y^2-x}$.

(b) The slope of the line tangent to the curve at $(1,-1)$ is $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right]_{x=1,y=-1} = \left[\frac{y-3x^2}{6y^2-x}\right]_{x=1,y=-1} = -\frac{4}{5}$.

Hence, the tangent line has equation $y+1 = -\frac{4}{5}(x-1)$, which simplifies (optional) to $y = -\frac{4}{5}x - \frac{1}{5}$. □

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