Midterm #2

Please print your name:

Besides the allowed calculator, no notes or tools of any kind are permitted.

There are 27 points in total.

Good luck!

Problem 1. (5 points) Compute the following derivatives. [No need to show work here.]

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}[x^7 - 2x^3 + e^2] =$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{\sqrt{x}} =$

(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln(x^3 - 1) =$

(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}[x^3 \tan^{-1}(x)] =$

(e) $\dfrac{\mathrm{d}}{\mathrm{d}x} e^{\cos(4x)} =$

Problem 2. (1 point) By the limit definition, $f'(7) =$.

Problem 3. (2 points) Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}(x+4)^x$.

Problem 4. **(3+1+1 points)** Consider the curve $x^2 + xy = e^y$. x

x

(a) Using implicit differentiation, determine $\frac{dy}{dx}$.

(b) Determine the line tangent to the curve at the point $(-1, 0)$.

(c) Determine the line normal to the curve at the point $(-1, 0)$.

Problem 5. **(3 points)** Use the graph below to fill in each entry of the grid with positive, negative or zero.

	$f(x)$	$f'(x)$	$f''(x)$
$x = -1$			
$x = 0$			
$x = 3$			

Problem 6. **(2 points)** Roughly sketch a differentiable function $f(x)$ with the following property.

(a) $f'(0) = 0$ but 0 is not a local extremum,   (b) $f'(0) < 0$ and $f''(0) > 0$.

x

x

x

x

Problem 4. **(3+1+1 points)** Consider the curve $x^2 + xy = e^y$.   [Show your work!]

(a) Using implicit differentiation, determine $\frac{dy}{dx}$.

(b) Determine the line tangent to the curve at the point $(-1, 0)$.

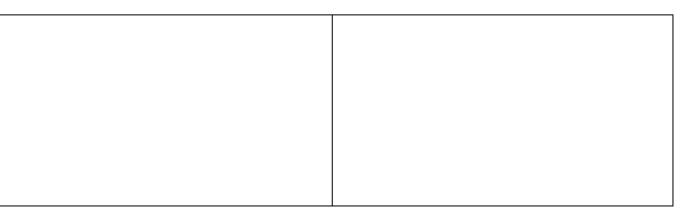(c) Determine the line normal to the curve at the point $(-1, 0)$.

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Problem 6. **(2 points)** Roughly sketch a differentiable function $f(x)$ with the following property.

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x

Problem 7. (3+1+1+1 points) Consider the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$.

(a) Determine all local extrema of $f(x)$.

(b) On which (open) intervals is $f(x)$ increasing?

(c) On which (open) intervals is $f(x)$ concave up?

(d) $f(x)$ has an inflection point at $x =$ [   ].

Problem 8. (3 points) Oil is leaking from a tanker and spreads in a circle whose area increases at a rate of 10 km^2/h. How fast is the radius of the spill increasing after 3 h?

(extra scratch paper)

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