Practice for Midterm #1

MATH 125 — Calculus 1
Tuesday, Feb 5

Please print your name:

Besides the allowed calculator, no notes or tools of any kind will be permitted.

Problem 1. Have another look at the homework, especially those problems that you struggled with.

Problem 2. Retake the three quizzes! (Versions with and without solutions are posted to our course website.)

Problem 3. Go through the lecture sketches (posted to our course website) and do the problems we did in class (ignore the solutions until you have solved the problem yourself).

Problem 4. Determine the following limits (or state that they don't exist).

(a) $\lim_{x\to -1} \frac{x+3}{x+1}$

(b) $\lim_{x\to\infty} \tan^{-1}(x)$

(c) $\lim_{x\to -\infty} \sin(x)$

(d) $\lim_{x\to -\infty} \frac{\sin(x)}{1+x^2}$

(e) $\lim_{x\to\infty} \frac{2+\sqrt{x}-2x^3}{5-x^3}$

(f) $\lim_{x\to 0} \frac{\sin(2x)}{\sin(x)}$

(g) $\lim_{x\to 0} \frac{\sin(2x)}{\sqrt{x}}$

(h) $\lim_{x\to\infty} \frac{\sin(2x)}{\sqrt{x}}$

Problem 5. Determine the following limits. [Show work!]

(a) $\lim_{x\to\infty} \left(x^2 - \sqrt{x^4+5x+1}\right)$.

(b) $\lim_{x\to\infty} \left(x^2 - \sqrt{x^4+5x^2+1}\right)$.

(c) $\lim_{x\to\infty} \left(x^2 - \sqrt{x^4+5x^3+1}\right)$.

Problem 6. For what values of a and b is $f(x) = \begin{cases} \sin(x)-a, & x<0, \\ a\sqrt{x+1}+b, & 0\leqslant x\leqslant 3 \\ x+2b, & x>3, \end{cases}$ continuous at every x? [Show work!]

Problem 7. [Show work!]

(a) Determine $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = x^2-3x$.

(b) Determine $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = \sqrt{2x+1}$.

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Problem 8. Let $f(x)$ be a complicated continuous function taking the following values:

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-1	2	4	1	-1	-3	4	4	1

How many solutions to the equation $f(x)=3$ can we guarantee? How about $f(x)=5$? Or $f(x)=4$?

Problem 9. Let $f(x)=x+\sqrt{\ln(x)}$.

(a) At which points is $f(x)$ continuous?

(b) Show that there is a number c in the interval $[1,5]$ such that $f(c)=3$.

Problem 10. Consider the function $f(x)=\begin{cases} x, & \text{if } x \leqslant -1, \\ x^2, & \text{if } -1<x\leqslant 0, \\ x\sin\left(\frac{1}{x^2}\right), & \text{if } x>0. \end{cases}$

(a) Find the limit $\lim_{x\to 0^+} f(x)$.

(b) At which points is $f(x)$ discontinuous?

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